Fill in the blanks. **SCORE: _____ / 4 PTS**

[a] Playa Spiaggia Beach has a straight coastline, with a lifeguard tower on the beach (not on the coastline).

A dog is running on the beach so that it is always exactly as far from the coastline as it is from the tower.

The dog's path is in the shape of a/an PARABOLA (2) (or part of it).

[b] The shape of the graph of the equation $3x^2 - 3x + 2y^2 - 2y - 1 = 0$ is a/an ELLIPSE (1).

[c] The shape of the graph of the equation $3x^2 + 3x - 3y^2 - 3y - 1 = 0$ is a/an HYPERBOLA (1).

Convert the polar equation $r = \dfrac{1}{3 + \sin\theta}$ to rectangular form. **SCORE: _____ / 6 PTS**

Simplify your answer so that there are no radicals, complex fractions, fractional exponents nor negative exponents.

$r(3 + \sin\theta) = 1$

$3r + r\sin\theta = 1$

$3r + y = 1$ (2)

$3\sqrt{x^2+y^2} + y = 1$ (2)

$3\sqrt{x^2+y^2} = 1 - y$

$9(x^2+y^2) = (1-y)^2$

$9x^2 + 9y^2 = 1 - 2y + y^2$ (1)

$9x^2 + 8y^2 + 2y - 1 = 0$ (1)

Find the vertices, foci and equations of the asymptotes of the hyperbola $3x^2 - 2y^2 - 12x - 20y - 14 = 0$. **SCORE: _____ / 6 PTS**

$3x^2 - 12x - 2y^2 - 20y = 14$

$3(x^2 - 4x) - 2(y^2 + 10y) = 14$

$3(x^2 - 4x + 4) - 2(y^2 + 10y + 25) = 14 + 12 - 50$

(1) $3(x-2)^2 - 2(y+5)^2 = -24$ (1)

$\dfrac{(y+5)^2}{12} - \dfrac{(x-2)^2}{8} = 1$ $(\tfrac{1}{2})$

CENTER $(2, -5)$ (1)

VERTICES $(2, -5 \pm 2\sqrt{3})$

$c^2 = 12 + 8 = 20$

$(\tfrac{1}{2})$ $c = 2\sqrt{5}$ $(\tfrac{1}{2})$

FOCI $(2, -5 \pm 2\sqrt{5})$

SLOPE $= \pm\sqrt{\dfrac{12}{8}} = \pm\sqrt{\dfrac{3}{2}}$

$= \pm\dfrac{\sqrt{6}}{2}$ (1)

$(\tfrac{1}{2})$

ASYMPTOTES $y + 5 = \pm\dfrac{\sqrt{6}}{2}(x - 2)$

A point has polar co-ordinates $(6, \frac{4\pi}{5})$. SCORE: _____ / 2 PTS

[a] Find another pair of polar co-ordinates for the point, using a positive value of r and a negative value of θ.

$(6, \frac{4\pi}{5} - 2\pi) = (6, \frac{-6\pi}{5})$ 1

[b] Find another pair of polar co-ordinates for the point, using a negative value of r and a positive value of θ.

$(-6, \frac{4\pi}{5} + \pi) = (-6, \frac{9\pi}{5})$ 1

Convert the rectangular equation $y = 2x - 3$ to polar form. Write r as function of θ, and simplify your answer. SCORE: _____ / 3 PTS

$r\sin\theta = 2r\cos\theta - 3$ 1

$r\sin\theta - 2r\cos\theta = -3$

$r(\sin\theta - 2\cos\theta) = -3$ 1

$r = \dfrac{-3}{\sin\theta - 2\cos\theta} = \dfrac{3}{2\cos\theta - \sin\theta}$ 1 EITHER FORM IS OK

Using the distance based definition (as shown in lecture for ellipses and parabolas), SCORE: _____ / 9 PTS
find the equation of the hyperbola with foci $(0, -5)$ such that the distances from the foci to any point on the hyperbola differ by 2.

$\sqrt{x^2+(y+5)^2} - \sqrt{x^2+(y-5)^2} = \pm 2$ 2

$\sqrt{x^2+(y+5)^2} = \pm 2 + \sqrt{x^2+(y-5)^2}$ 1

$x^2+y^2+10y+25 = 4 \pm 4\sqrt{x^2+(y-5)^2} + x^2+y^2-10y+25$ 2

$20y - 4 = \pm 4\sqrt{x^2+(y-5)^2}$

$5y - 1 = \pm\sqrt{x^2+(y-5)^2}$ 1

$25y^2 - 10y + 1 = x^2 + y^2 - 10y + 25$ 1

$24y^2 - x^2 = 24$ 1

$y^2 - \frac{x^2}{24} = 1$ 1